Moment of Inertia

Objective

The objective of this experiment is to use the standard (linear) and rotational forms of Newton's second law to empirically obtain the moment of inertia of a heavy disk.

Materials

1. C-clamp
2. Caliper
3. Hooked 1000g mass
4. Moment of inertia wheel
5. Pasco 550 Interface
6. Smart Pulley with chrome rod
7. Vinyl-jawed spring clamp

Introduction

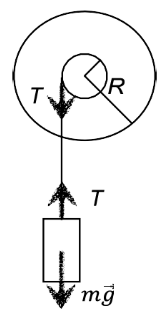

Figure 1:*Free Body Diagram for the setup*

In this experiment we will use both the standard form of Newton's second law, $\vec{F}_{net} = m\vec{a}$ and the rotational form, $\tau_{net} = I\alpha$, where τ is the torque, α is angular acceleration, and I is moment of inertia. Our aim will be to obtain the moment of inertia of a wheel by measuring the acceleration produced when a mass is suspended by a string that is wrapped vertically around the horizontal axle of the wheel.

The diagram in Figure 1 represents a wheel of radius R on an axle of radius r. As mass m descends it causes the wheel to rotate at an angular velocity ω, which increases at rate α (measured in radians/s^2). Applying Newton's second law (letting downward be the positive y-direction) to m we have

$$mg - T = ma$$

where T is the tension in the string, and a is the magnitude of the downward acceleration of the mass m. (We assume the mass of the string is negligible.)

In this experiment we will be able to measure a; therefore, we can obtain T

$$T = m(g - a) \quad (1)$$

Applying the rotational form of the second law to the wheel (letting the positive direction be that of counterclockwise rotation) we have

$$Tr - \tau_f = I\alpha$$

where Tr is the torque due to the string (force T times lever arm r), and τ_f is the torque due to friction f. As a first approximation let us assume that the torque due to friction is negligible. The better the bearings the more nearly valid this assumption will be. Then

$$I \approx Tr/\alpha$$

α and a are related by $a = r\alpha$. Therefore,

$$I \approx Tr^2/a \quad (2)$$

Using T from equation (1) and measured values of a and r, we can calculate I from equation (2).

Our technique for measuring a will involve having a ten-spoke pulley roll against the axle while the computer records the times at which a photogate beam is broken by the spokes. Then the computer will obtain a as the slope of the best straight line fit to a graph of velocity vs time. Because the pulley rolls against the axle, the tangential accelerations of points on their rims will be the same, and this tangential acceleration will also be the acceleration, a, of the mass on the string.

Procedure

Equipment Setup

Clamp the wheel bracket to the top of the corner of the table by means of the C-clamp. (Please be careful, the wheel is a potential toe crusher!) Thread one end of a string about one meter long through the hole in the axle, and wind the string tightly, being careful to avoid overlaps— keep it one layer thick. Put a loop in the end for use in hanging a weight. Use the spring clamp to hold the chrome rod of the Smart Pulley in such a way that the pulley will roll against the axle on the side of the wheel without the string (see Figure 2).

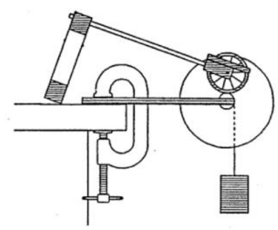

Figure 2: The Setup

Part I. Computer Measurements

Connect the photogate to digital channel #1 of the Pasco interface. On the desktop double click on “Pasco Capstone”. In the "Tools" windows panel on the left side of the screen, click on "Hardware Setup".

Click the yellow circle around Channel 1, scroll down the list of sensors, and choose “Photogate with Pulley”. To get a velocity-time graph, double click on “Graph” (in the “Displays” window panel on the right side of screen) then click on “Select Measurement” on the vertical axis of your scope and choose “Linear Speed”. (This should give you a velocity-time graph). While holding the wheel firmly, hook one kilogram on the end of the string. Press the “Record” button to activate the timer. Release the wheel. Press “Stop” to stop the timer. Immediately after stopping the timer, stop the wheel so that it will not tangle the string nor hit the floor. You should see that data was taken on your graph. To get a better look at your data, you may need to use either the auto-scale button or zoom into your vertical axis. Then use the “Data Highlighter” tool to select the linear portion of your graph. With that area highlighted, go to the top your graph, and click on the “Curve Fits” tool.

From the drop-down, choose “Linear”. This should give you a small window of information about your linear curve-fit. Recall that acceleration is the slope of a velocity vs time graph. The slope of your graph should appear in your linear fit window (“m =”... in units of m/s^2). Use this acceleration together with other necessary measurements in equation (2) to calculate the moment of inertia, I, of the wheel in units of $kg{\cdot}m^2$.

Part II. Another Look at Friction

Let us reconsider our assumption that friction is negligible. Remove the string from the wheel. Spin the wheel by hand, then press "Record" to activate the timer. Stop the timer after a few seconds and proceed as before to obtain the acceleration. This time you have only the torque due to friction. It should be related to the measured acceleration by

$$\tau_f = I\alpha = I\frac{a}{r}$$

Use your previously measured value of I in calculating τ_f. Is the frictional torque negligible compared to the string torque, Tr? If not, does this mean that your value for I is too large or too small? Can you estimate the approximate size of the error? Can you use your knowledge of the torque due to friction to obtain a more accurate value of the moment of inertia, I?